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16. Abstract

An explicit two-step finite-difference scheme with second-order accuracy in the determination of the spatial variables is proposed for solving the problems of flows past bodies. A distinguishing feature of the finite-difference scheme is the small number of arithmetical operations required for calculation of parameters at each of its predictor-corrector steps.

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AN EXPLICIT FINITE-DIFFERENCE SCHEME FOR SOLVING THE PROBLEMS OF FLOWS PAST BODIES BY THE FINITE DIFFERENCE METHOD /119*

A. N. Minaylos

1. Explicit schemes, in view of their simplicity, are used widely in solving gas-dynamic problems. As the number of independent variables is increased, the volume of computational work in calculating one node of a mesh, as well as the number of nodes as such. This leads to large outlays of computer time, therefore the problem of setting up an effective finite-difference scheme becomes essential. This scheme must be simple, while satisfying the requirements of approximation and stability.

P. D. Lax's well-known scheme is one of the simplest. Its generalizations, weakening the effect of artificial viscosity by using parameter α, were examined in references /1/ and /2/. Let L stand for this scheme /1/. Its disadvantages are as follows.

When there is a large coefficient of viscosity (α is close to zero and the scheme is close to P. D. Lax's scheme), the solution is rapidly ascertained, but contains a large error caused by the presence of viscous terms; at this coefficient of viscosity (α is close to unity) the solution becomes unstable. Here we consider the linear relation between steps of the network τ and h in terms of time and spatial variables, since given the function $\tau \sim h^2$ yielding a stable computation when $\alpha = 1$, the step in time is too small and the network is not economically advantageous. Computation in reference /1/ with step $\tau \sim h$ and $\alpha = 1$ was stable for a mesh with a small number of cells (6 x 2) and with the stabilizing influence of a shock wave. Another L - W scheme examined in /1/ belongs to the class investigated by P. D. Lax and B. Vendrov. It was used in the

* Numbers in the margin indicate pagination in the foreign text.

study /3_7 and gives more exact results than does scheme L. Fluctuations in the shock wave front during the finding of the solution by this scheme are small in amplitude, therefore the field is less perturbed in the computational region and the solution-finding process proceeds more rapidly. However, when parameters are computed at each node of the mesh under the scheme, considerably more operations are required than by scheme L, since the second derivatives in time are determined by differentiation of the system of equations with respect to all independent variables. The cumbersomeness of the expressions intensifies when spatial flows are calculated.

This article deals with selecting a difference scheme such that, by preserving the advantages of the L - W scheme, would be less cumbersome, and the number of arithmetical operations at each step in it would be much fewer.

2. The proposed scheme N is a simplified variant of the scheme used in /4_7. The scheme in /4_7 -- let us call it R -- is a modification of R. D. Richtmyer's scheme. This is a two-step scheme of the predictor-corrector type, of second-order accuracy.

Scheme N differs from scheme R in that the spatial derivative /120 at the second corrector-step, with intermediate layer, is taken only along one direction, while the derivatives of the old layer are used in the other.

Let us write out scheme N for the case of two spatial variables s and n. We represent the system of gas-dynamic equations as

$$U_t' = F(U, U_s', U_n'); \tag{1}$$

where U is the vector of the unknown functions, F is the operator of the right-hand sides of the system, and F_h is the difference operator approximating the operator F with approximation order 2. As in scheme R, the calculation proceeds in two stages.

First one calculates the values of the unknown functions at an \auxiliary layer coinciding with the layer t + τ. In contrast to the scheme R, these values are determined only at two points lying on the same coordinate line with the central point (i, j). Thus, the scheme is anisotropic and so it loses its commonality in the sense that its application necessitates selecting a specific direction, that is, some knowledge of the solution of the problem. In the problem of flow past a blunt body the points in the intermediate layer are taken along the coordinate line s = const, that is, along some arckdirected from the body to the shock wave. It is precisely in this direction, as experience shows, that the main fluctuations in the solution occur in the process of arriving at the solution. The correctness of this approach is confirmed by practical calculations. The values at the auxiliary points (i, j + 1/2) and i, j - 1/2) are determined by the scheme L with $\alpha = 0$:

$$\left.\begin{aligned} U_{i,j-1/2} &= \frac{U_{i,j}+U_{i,j-1}}{2};\\ (U_{i,j-1/2})'_s &= \frac{(U_{i+1,j}-U_{i-1,j})+(U_{i+1,j-1}-U_{i-1,j-1})}{4h_s};\\ (U_{i,j-1/2})'_n &= \frac{U_{i,j}-U_{i,j-1}}{h_n};\\ \overline{U}^{\tau}_{i,j-1/2} &= U_{i,j-1/2}+\tau F_h(U,\ U'_s,\ U'_n)_{i,j-1/2}. \end{aligned}\right| \tag{2}$$

The subscript t is omitted in the expressions.

Formulas for the point (i, j + 1/2) were obtained from Eqs. (2) by adding unity to the subscript j.

Then one determines the values of F_h at the point (i, j) in the "old" layer t and at the intermediate layer t + τ.

Let us use the following formula to determine the values in the new layer t + τ:

$$U^{2}_{i,j} = U_{i,j} + \tau[\beta F_h(U, U'_s, U'_n)_{i,j} + (1-\beta) F_h(\bar{U}, \bar{U}'_s, \bar{U}'_n)_{i,j}]; \tag{3}$$

here

$$\bar{U}_{i,j} = \frac{\bar{U}_{i,j+1/2} + \bar{U}_{i,j-1/2}}{2};$$

$$(\bar{U}_{i,j})'_s = (U_{i,j})'_s = \frac{U_{i+1,j} - U_{i-1,j}}{2h_s};$$

$$(\bar{U}_{i,j})'_n = \frac{\bar{U}_{i,j+1/2} - \bar{U}_{i,j-1/2}}{h_n};$$

$$(U_{i,j})'_n = \frac{U_{i,j+1} - U_{i,j-1}}{2h_n}.$$

In scheme L we must use the block of calculation of /121
operator F_h three times (with allowance for the values of $\bar{U}_{i,j-1/2}$ retained from the calculation of the preceding point (i, j - 1)). In the variant with $\beta = 0$, we must use the block for calculating operator F_h twice at one point.

Below is given a comparative table of the amount of computational work at one node of the mesh for different schemes (the number of operations for a single use of the F_h operator calculation block is taken as unity):

TABLE 1

	L	L - W	R	N
Two spatial derivatives	1	6--8	4--5	2--3
Three spatial derivatives	1	12--14	6--7	2--3

3. Analysis of the simplified system shows that when $\tau \sim h$ the scheme N is unstable. The instability is caused by the terms

determining the derivatives with respect to the direction s in layer t. To stabilize the solution, let us use the smoothing process equivalent to introducing artificial viscosity. We note that at the predictor step the method now contains terms with artificial viscosity in P. D. Lax's scheme.

Let us look at the effect of different laws of smoothing on the stability of the scheme L for a simple model equation:

$$u_t + au_x = 0 \quad (a > 0). \tag{4}$$

Let us represent the scheme as $u_m^{\tau} = \tilde{u} - \frac{a\tau}{2h}(u_{m+1} - u_{m-1})$.

The results of analysis by the spectral method are given in Table 2:

TABLE 2

α	Smoothed value of u	Stability conditions
0	$\frac{1}{2}(u_{m+1} + u_{m-1})$	$\tau \leqslant \frac{h}{a}$
$\frac{1}{3}$	$\frac{1}{3}(u_{m+1} + u_m + u_{m-1})$	$\tau \leqslant \frac{2\sqrt{2}}{3a} h$
$\frac{2}{3}$	$\frac{1}{6}(u_{m+1} + 4u_m + u_{m-1})$	$\tau \leqslant \frac{\sqrt{5}}{3a} h$
$\frac{4}{5}$	$\frac{1}{10}(u_{m+1} + 8u_m + u_{m-1})$	$\tau \leqslant \frac{3}{5a} h$
1	u_m	Unstable

The limiting values of τ decrease with weakening of smoothing.

The example in the second row of Table 2 corresponds to smoothing by the least-squares method, with the linear smoothing polynomial constructed at three nodes being specified. We can see without difficulty that all these linear forms of smoothing retain values lying along a straight line and convert the second-order curve into a curve of second order with a shift proportional to h^2.

Actually, the formula /122

$$\tilde{u} = \frac{1}{3}(u_{m+1} + u_m + u_{m-1}) = u_m + \frac{1}{3}[(u_{m+1} - u_m) + (u_{m-1} - u_m)]$$

converts line mh into itself, and the curve m^2h^2 into $m^2h^2 + h^2/3$.

The presence of this shift when the form of the shock wave being smoothed (close to a second-order curve) leads to a large distortion of results, therefore smoothing based on interpolation employing a quadratic polynomial was used. In this case the least-squares method, for the four nodes used, yields the smoothing formula given below for Eq. (5):

$$\tilde{u} = u_m - \frac{3}{20}[u_{m+2} - u_{m-1} - 3(u_{m+1} - u_m)].$$

The scheme is stable when $\tau \leqslant 0.288\ h/a$. Smoothing of this type in fact was used to increase the stability of scheme N. In the case of three spatial variables smoothing is conducted separately at each meridional half-plane by the formula

$$\begin{aligned}\tilde{u}_{i,j} = u_{i,j} - \frac{3}{20}\{\chi[u_{i,j+2} - u_{i,j-1} - 3(u_{i,j+1} - u_{i,j})] + \\ + (1-\chi)[u_{i+2,j} - u_{i-1,j} - 3(u_{i+1,j} - u_{i,j})]\},\end{aligned}$$

where $0 \leqslant \chi \leqslant 1$.

Smoothing is used not at each step τ, but after a certain number of steps $k > 1$. Here the stability condition becomes more rigorous (when $k = 2$, by scheme (5) $\tau \leqslant (0.25/1)h$).

The values of k and χ are selected empirically for the individual variants. Usually $k = 10$-20, $\chi = 0.5$.

4. Scheme N was used in constructing, jointly with A. P. Bazzhin and S. V. Pirogova, a program for computing flow of a

supersonic stream past a blunt body /5_7. The parameters at the boundaries were calculated by the method of characteristics close to the scheme of D. Moretty /3_7. The internal nodes of the solution domain were computed under scheme N.

A large series of calculations of two-dimensional flows in an ideal gas were made under this program (with constant ratio of specific heat capacities), as well as in equilibrium-dissociating air. Flow past spheres, ellipsoids of rotation of right and elliptical cylinders, and bodies of revolution in which the equation of the generatrix was in the form of a power monomial was calculated.

Experience in use of the program /5_7 - /7_7 confirmed that it is effective with satisfactory accuracy (errors of the order of 2-4 percent). Below several results are presented.

Fig. 1 shows the velocity at the surface of ellipsoids of revolution with ratio of axes a/b varying from zero (flat face) to two. The velocity is given with respect to the limiting velocity and is plotted as a function of length of ellipsoid arc; the radius of the middle of the ellipsoid (semiaxis b) is taken as the unit of linear dimension. The curve in the case a = 0 is obtained by the method of integral relations.

In Fig. 2 a comparison is made of the patterns of shock waves, streamlines, and lines of constant values of M number for flow past elliptical cylinders with a ratio of ellipse axes equal to unity (right cylinder) and 1.5. The results correspond to the values $\kappa = 1.4$ and $M_\infty = 1.5$. Calculation by the finite-difference method for small supersonic M_∞ numbers is difficult owing to the poor ascertaining of the solution (perturbations in the flow field are weakened). In addition, we know that the accuracy of the results of calculating plane flows is lower than for axi-symmetric flows, owing to the greater distances from the body surface to the shock wave. Therefore Fig. 2 illustrates the results of "difficult" calculations. The deviation of the values of Bernoulli's integral from the exact value does not exceed 1.5 percent for these results. However, for small M_∞ numbers this integral is usually computed to high accuracy, and the accuracy of its computation is not a sufficient condition for the accuracy of the results. Errors in the calculation do not exceed 3 percent, while the velocity of the shock wave at the end of the computation, that is, the degree of "arrival" at the variant is not more than $0.02\ V_{max}$.

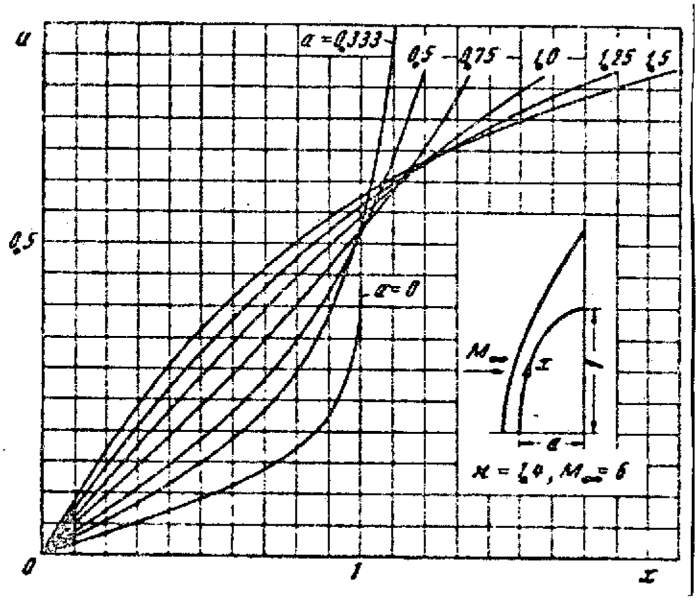

Fig. 1.

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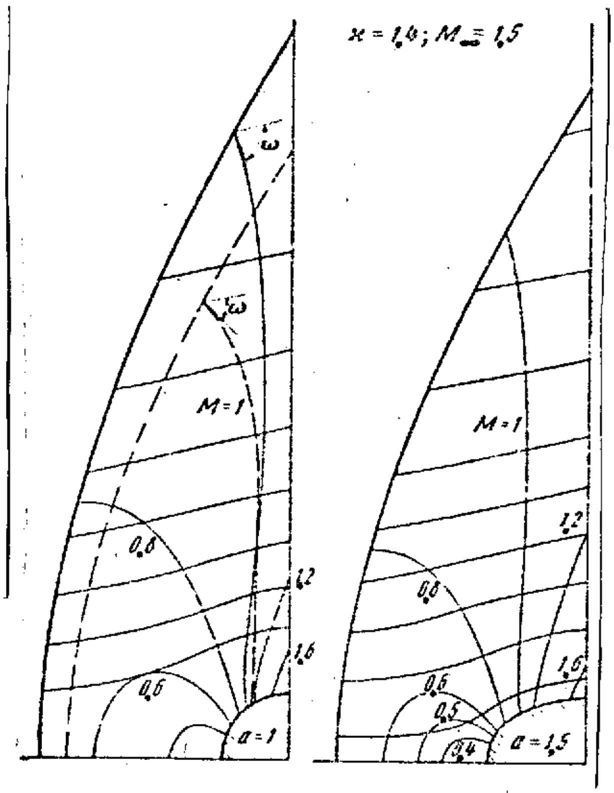

Fig. 2.

It is precisely the criterion of accuracy for the wave velocity that requires extended computation at low supersonic velocities, by increasing the number of steps four-five times with respect to the time variable. In the reference [8], the results of which are shown in Fig. 2 with a dashed line, evidently this criterion was not considered. The flow at $M_\infty = 1.5$ was not established, and the results contain large errors. For comparison, it is sufficient to examine the angle ω between the streamline and

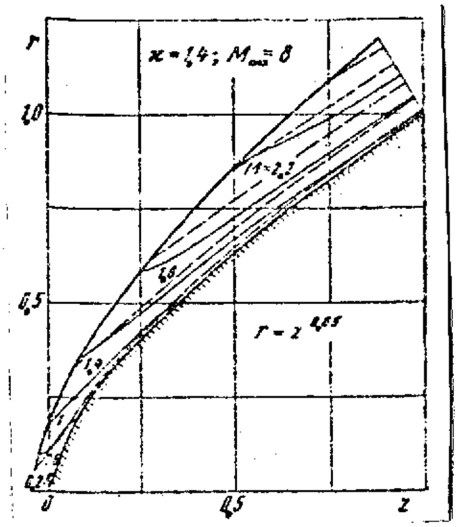

Fig. 3

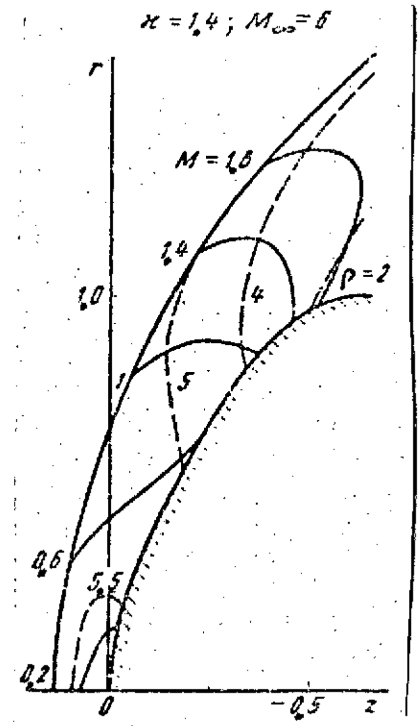

Fig. 4.

the sonic line at a point on the shock wave (see Fig. 2). The exact value obtained analytically is $\omega \approx 96^\circ$; the results of this present study is $\omega = 89^\circ$; and the results of reference /8_7 is approximately 60°. To enhance the accuracy of ω obtained in this study, requires more prolonged computation and possibly finer subdivision of the computational domain in the direction normal to the body.

V. I. Blagosklonov calculated flow past bodies of revolution /124
with generatrix in the form of a power monomial $r = z^m$ $(1/2 \leqslant m \leqslant 1)$. In Fig. 3 is shown the flow field for the variant with values $m = 0.65$, $\kappa = 1.4$, and $M_\infty = 8$. The length of the lines M = const is typically large compared with blunter bodies.

Fig. 4 shows the form of a shock wave, isomach, and isochor for a body of revolution whose generatrix is given by the formula

$$r = \frac{1}{1+\left(\frac{z}{z_l}\right)^n} \operatorname{tg}\alpha \sqrt{z^2 - c_1^2} + \frac{\left(\frac{z}{z_l}\right)^n}{1+\left(\frac{z}{z_l}\right)^n}\left(a_2 z^2 + b_2 z + c_2\right).$$

Here

$$a_1 = \frac{R_1}{\operatorname{tg}^2 \alpha}; \quad a_2 = -\frac{1}{2R_2};$$

$$b_2 = \frac{z_l}{R_2} + \frac{z_l \operatorname{tg} \alpha}{\sqrt{z_l^2 - a_1^2}};$$

$$c_2 = 1 - \frac{R_2}{2} b_2^2.$$

Calculations of flow for bodies of this form were made by A. P. Kosykh. In these calculations use was made of the nonuniform subdivision of the computational domain in the physical plane in the direction along the generatrix of the body (along the x axis).

Corresponding to the variant shown in Fig. 4 are the following values of the parameters: for body shape -- $R_1 = 0.6$, $R_2 = 0.2$, $\alpha = 60^\circ$, $n = 100$, and $z_\ell = 0.5045$; and for incident flow -- $\kappa = 1.4$ and $M_\infty = 6$.

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